

# Sign of the refractive index in a gain medium with negative permittivity and permeability

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We show how the sign of the refractive index in any medium may be derived using a rigorous analysis based on Einstein causality. In particular, we consider left-handed materials, i.e., media that have negative permittivities and permeabilities at the frequency of interest. We find that the consideration of gain in such media can give rise to a positive refractive index. © 2006 Optical Society of America

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## 1. INTRODUCTION

Crucial to the treatment of electromagnetic wave propagation is the connection between the signs of the electric permittivity  $\tilde{\epsilon}$  and the magnetic permeability  $\tilde{\mu}$  and that of the refractive index. The wave vector  $\tilde{k}(\omega)$  (and ultimately the index) is related to the square root of  $\tilde{\epsilon}(\omega)\tilde{\mu}(\omega)$ , which leaves the sign ambiguous. One way to determine the sign of the refractive index is by a numerical simulation of Maxwell's equations (see Refs. 1 and 2): From the wave evolution with proper initial conditions, the sign of the refractive index can be inferred. However, analytical justification of the sign of the refractive index has the advantages of generality and physical insight. In prior work, this has been based on the assertion that the directions of energy flux and the group velocity must point away from the radiation source (see Refs. 3–6). From this assertion, one may conclude that any medium for which the phase velocity is oppositely directed to the energy flux or group velocity will exhibit negative refraction, including the lossless systems for which both the real part of the electric permittivity  $\epsilon$  and the real part of the magnetic permeability  $\mu$  are simultaneously negative<sup>3,4</sup> and the lossy systems where these values need not be simultaneously negative.<sup>5,6</sup> However, the assertion itself is not true in general. It has been known for some time that in certain systems (both passive and active) the group velocity can actually point toward the source. This has been observed experimentally and can be understood as a pulse-reshaping phenomenon (Ref. 7 and references therein). It also has been pointed out that it is the front velocity that must satisfy the requirement of Einstein causality.<sup>7–9</sup> The front velocity must therefore point away from the source. The directions of phase velocity, group velocity, and energy flux then follow and can point in either direction depending on the details of the material response. Our discussion relies equally on (i) the implications of causality for the sign of the index and (ii) the introduction of gain in the system. Although our formalism is general, we focus on left-handed materials (LHMs),<sup>10</sup> i.e., media that have  $\epsilon, \mu < 0$  at the frequency of interest,<sup>3</sup> and we show that some LHMs give rise to positive refraction.

## 2. DETERMINING THE SIGN OF THE REFRACTIVE INDEX

We employ a rigorous analysis of causal wave propagation as first discussed by Sommerfeld and Brillouin.<sup>11</sup> The wave equations for a homogeneous isotropic linear medium in a source-free region are

$$\begin{cases} \nabla^2 \tilde{\mathbf{E}}(\mathbf{r}, \omega) = -\omega^2 \tilde{\epsilon}(\omega) \tilde{\mu}(\omega) \tilde{\mathbf{E}}(\mathbf{r}, \omega) \\ \nabla^2 \tilde{\mathbf{B}}(\mathbf{r}, \omega) = -\omega^2 \tilde{\epsilon}(\omega) \tilde{\mu}(\omega) \tilde{\mathbf{B}}(\mathbf{r}, \omega) \end{cases} \quad (1)$$

$\tilde{\mathbf{E}}(\mathbf{r}, \omega)$  and  $\tilde{\mathbf{B}}(\mathbf{r}, \omega)$  are the complex Fourier transforms of the corresponding real fields  $\mathbf{E}(\mathbf{r}, t)$  and  $\mathbf{B}(\mathbf{r}, t)$ , where the Fourier transforms in Eqs. (1) are defined as

$$\tilde{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \exp(i\omega t) dt. \quad (2)$$

The vector equations have a common form for each Cartesian component and can be simplified to

$$\frac{\partial^2 \tilde{g}(z, \omega)}{\partial z^2} = -\omega^2 \tilde{\epsilon}(\omega) \tilde{\mu}(\omega) \tilde{g}(z, \omega), \quad (3)$$

if we consider a general plane wave propagating in the  $+z$  direction. Equation (3) has the time-domain Green's function solutions

$$\begin{aligned} g(z, t) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{g}(z, \omega) \exp(-i\omega t) d\omega \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp[i(\tilde{k}(\omega)z - \omega t)] d\omega. \end{aligned} \quad (4)$$

$\tilde{k}(\omega)$  is in general not single valued because of the branch cuts created when we take the square root, and Einstein causality is needed to determine the correct branch of  $\tilde{k}(\omega)$ . More precisely, with  $\tilde{\epsilon}(\omega) = |\tilde{\epsilon}(\omega)| \exp(i\phi_\epsilon)$  and  $\tilde{\mu}(\omega) = |\tilde{\mu}(\omega)| \exp(i\phi_\mu)$ , the two branches are  $\tilde{k}(\omega) = \pm \omega \sqrt{|\tilde{\epsilon}(\omega)| |\tilde{\mu}(\omega)| \exp[i(\phi_\epsilon + \phi_\mu)/2]}$ , with a corresponding refractive index  $\tilde{n}(\omega) = \pm c \sqrt{|\tilde{\epsilon}(\omega)| |\tilde{\mu}(\omega)| \exp[i(\phi_\epsilon + \phi_\mu)/2]}$ .

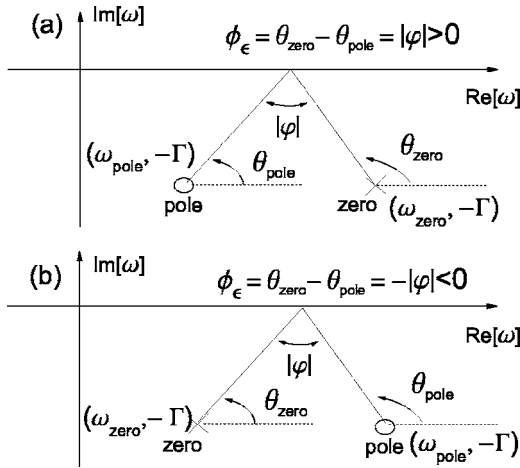


Fig. 1. Zero-pole pairs generated by (a) a noninverted Lorentz oscillator ( $F > 0$ ) and (b) an inverted Lorentz oscillator ( $F < 0$ ). Only the poles and zeros for  $\text{Re}[\omega] > 0$  are indicated, as the structures are symmetric with respect to  $\text{Re}[\omega] = 0$ .

Using Titchmarsh's theorem,<sup>12,13</sup> it can be shown that for any physical system  $\tilde{k}(\omega)$  is analytical in the upper half of the complex  $\omega$  plane and on the real  $\omega$  axis. Additionally, the two branches of  $\tilde{k}(\omega)$  have the limits of  $+(\omega/c)$  and  $-(\omega/c)$ , respectively, as  $|\omega| \rightarrow \infty$ .<sup>14</sup> Evaluating the integral in Eq. (4) through contour integration in the complex  $\omega$  plane<sup>7,8,11,15</sup> then reveals that the two branches  $\tilde{n}(\omega) = +c\sqrt{|\tilde{\epsilon}(\omega)||\tilde{\mu}(\omega)|}\exp[i(\phi_\epsilon + \phi_\mu)/2]$  and  $\tilde{n}(\omega) = -c\sqrt{|\tilde{\epsilon}(\omega)||\tilde{\mu}(\omega)|}\exp[i(\phi_\epsilon + \phi_\mu)/2]$ , respectively, correspond to the retarded Green's function,  $g(z, t) = 0$  for  $t < (z/c)$ , and the advanced Green's function,  $g(z, t) = 0$  for  $t < -(z/c)$ . In other words, the two Green's functions have a front velocity of  $\pm c$ , respectively. Einstein causality requires that the front velocity has to point away from the source; thus only the retarded Green's function with front velocity  $+c$  is physical<sup>16</sup> and  $\tilde{n}(\omega)$  is then uniquely determined to be  $\tilde{n}(\omega) = +c\sqrt{|\tilde{\epsilon}(\omega)||\tilde{\mu}(\omega)|}\exp(i\phi_n)$ , where  $\phi_n = (\phi_\epsilon + \phi_\mu)/2$ .

The zero-pole structure of  $\tilde{\epsilon}(\omega)$  and  $\tilde{\mu}(\omega)$  is guaranteed to be symmetric with respect to  $\text{Re}[\omega] = 0$  by the reality of the electromagnetic field, the polarization, and the magnetization.<sup>17</sup> This symmetry leads to the relation  $\tilde{n}(-\omega) = \tilde{n}^*(\omega)$  with  $\omega$  on the real axis. We therefore only need to investigate the properties of  $\tilde{n}(\omega)$  for  $\omega > 0$  in the analysis that follows.

### 3. $\tilde{\epsilon}$ AND $\tilde{\mu}$ IN A TWO-LEVEL SYSTEM

We now express the permittivity and the permeability in terms of their underlying molecular polarizabilities and magnetizabilities. The expressions are equivalent to those obtained from a Lorentz oscillator model.

The isotropic component of the complex polarizability  $\tilde{\alpha}$  for a two-level system is given by

$$\tilde{\alpha} = \frac{2\omega_{mg}|g|\mathbf{p}|m\rangle|^2}{3[\omega_{mg}^2 - (\omega + i\Gamma)^2]}, \quad (5)$$

where  $|g\rangle|\mathbf{p}|m\rangle$  is the electric dipole transition moment between the ground state  $g$  and the excited state  $m$ .  $\omega_{mg}$  is

the angular transition frequency and  $\Gamma$  is the half-width at half-maximum of the Lorentzian spectrum ( $\Gamma > 0$ ).

In media for which

$$\tilde{\epsilon} = \epsilon_0 \left( 1 + \frac{N\tilde{\alpha}}{\epsilon_0} \right), \quad (6)$$

the electric permittivity is given by

$$\tilde{\epsilon} = \epsilon_0 \left\{ 1 + \frac{2\omega_{mg}N|g|\mathbf{p}|m\rangle|^2}{3\epsilon_0[\omega_{mg}^2 - (\omega + i\Gamma)^2]} \right\}, \quad (7)$$

where  $N$  is the number density, and where we can define

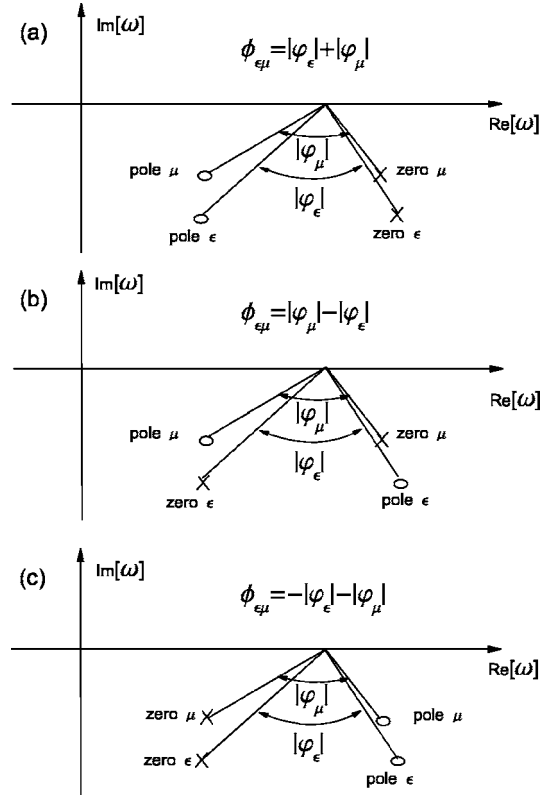


Fig. 2. Three of the possible four different types of zero-pole pairs are shown for LHM: (a)  $F > 0$  and  $G > 0$ , (b)  $F < 0$  and  $G > 0$ , (c)  $F < 0$  and  $G < 0$ . The one not shown corresponds to (b) with  $\epsilon$  and  $\mu$  exchanged.

Table 1. Four Types of LHM<sup>a</sup>

LHM	$\phi_\epsilon$	$\phi_\mu$	$\text{Re}[\tilde{n}]$
Type I			
$F > 0$	$(\pi/2) < \phi_\epsilon < \pi$	$(\pi/2) < \phi_\mu < \pi$	$< 0$
$G > 0$			
Type II			
$F < 0$	$-\pi < \phi_\epsilon < -(\pi/2)$	$(\pi/2) < \phi_\mu < \pi$	$> 0$
$G > 0$			
Type III			
$F > 0$	$(\pi/2) < \phi_\epsilon < \pi$	$-\pi < \phi_\mu < -(\pi/2)$	$> 0$
$G < 0$			
Type IV			
$F < 0$	$-\pi < \phi_\epsilon < -(\pi/2)$	$-\pi < \phi_\mu < -(\pi/2)$	$< 0$
$G < 0$			

<sup>a</sup> $F$  and  $G$  are Defined in Eqs. (9) and (10) and  $\phi_n = (\phi_\epsilon + \phi_\mu)/2$ .

$$F = \frac{2\omega_{mg}N|\langle g|\mathbf{p}|m\rangle|^2}{3\epsilon_0}. \quad (8)$$

The magnetic permeability may similarly be written in terms of the magnetizability. The following expressions for the electric permittivity  $\tilde{\epsilon}(\omega)$  and the magnetic permeability  $\tilde{\mu}(\omega)$  may thus be obtained as

$$\tilde{\epsilon}(\omega) = \epsilon_0 \left[ 1 + \frac{F}{\omega_{\text{pole } \epsilon}^2 - (\omega + i\Gamma)^2} \right] = \epsilon_0 \frac{(\omega + i\Gamma)^2 - \omega_{\text{zero } \epsilon}^2}{(\omega + i\Gamma)^2 - \omega_{\text{pole } \epsilon}^2}, \quad (9)$$

$$\tilde{\mu}(\omega) = \mu_0 \left[ 1 + \frac{G}{\omega_{\text{pole } \mu}^2 - (\omega + i\Gamma)^2} \right] = \mu_0 \frac{(\omega + i\Gamma)^2 - \omega_{\text{zero } \mu}^2}{(\omega + i\Gamma)^2 - \omega_{\text{pole } \mu}^2}, \quad (10)$$

where  $\omega_{\text{zero } \epsilon}^2 = \omega_{\text{pole } \epsilon}^2 + F$ , and where similarly  $\omega_{\text{zero } \mu}^2 = \omega_{\text{pole } \mu}^2 + G$ .  $F$ ,  $G$ , and  $\Gamma$  are all taken to be real. A system in the ground state corresponds to  $F, G > 0$  and an inverted system has  $F, G < 0$ . The permittivity of the vacuum is  $\epsilon_0$  and its permeability is  $\mu_0$ .

From Eq. (9) it follows that the structures of the zeros and poles determine  $\phi_\epsilon$ . In the case of a noninverted Lorentz oscillator, the contribution from the zero-pole pair yields a positive  $\phi_\epsilon$ , whereas for the inverted system,  $\phi_\epsilon$  is negative. This is shown in Fig. 1 where it is seen that the contribution from the zero-pole pair yields a positive  $\phi_\epsilon$  for a noninverted Lorentz oscillator, whereas for the inverted system,  $\phi_\epsilon$  is negative.<sup>18</sup> Similar conclusions may be drawn for  $\phi_\mu$  from Eq. (10).

The angles  $\phi_{\epsilon\mu} = (\phi_\epsilon + \phi_\mu)$  and  $\phi_n = \phi_{\epsilon\mu}/2$  are determined by the zeros and poles of both  $\tilde{\epsilon}$  and  $\tilde{\mu}$ . For the subsequent discussion, we consider the cases for which the sign of  $F$  and  $G$  in Eqs. (9) and (10) can independently vary. We suppose that this may, at least in principle, be achieved in a multicomponent system or in certain multilevel systems. Considering a two-component system, we assume that in a certain frequency range the electric permittivity is dominated by one molecular species whereas the magnetic permeability is dominated by the other species. If only one of the species is inverted, then we may have  $F$  and  $G$  of opposite sign. Similarly, in an inverted one-component multilevel system there may be nearby lying states that have electric dipole allowed, but magnetic dipole forbidden transitions, such that the polarizability and magnetizability, which respectively underlie  $\tilde{\epsilon}(\omega)$  and  $\tilde{\mu}(\omega)$ , are of opposite sign. If for these systems we describe  $\tilde{\epsilon}(\omega)$  and  $\tilde{\mu}(\omega)$  by an effective Lorentz oscillator model, then we have the following possible combinations:  $F, G > 0$ ,  $F$  and  $G$  of opposite sign, and  $F, G < 0$ . Figure 2 shows three of the four possible combinations described above.

#### 4. FOUR TYPES OF LEFT-HANDED MATERIAL

A left-handed medium requires  $F$  in Eq. (9) and  $G$  in Eq. (10) to be large compared with  $\Gamma$ , such that  $|\phi_\epsilon|$  and  $|\phi_\mu|$  exceed  $(\pi/2)$ , and such that both  $\text{Re}[\tilde{\epsilon}(\omega)]$  and  $\text{Re}[\tilde{\mu}(\omega)]$  are negative for some range of  $\omega$ . However, since  $\phi_n$

$= (\phi_{\epsilon\mu}/2) = (\phi_\epsilon + \phi_\mu)/2$  and since  $\phi_\epsilon$  and  $\phi_\mu$  can be of either sign,  $|\phi_n|$  can be smaller than  $(\pi/2)$  depending on the specific form of the zero-pole pair structure. Hence, a LHM does not necessarily need to have a negative refractive index. Table 1 lists the four possible combinations. To our knowledge only the passive LHM with a negative refractive index (type I) has been discussed previously (see, for instance, Refs. 19 and 20). In fact, additional types of LHM are possible if the materials are allowed to have multiple resonances. In this case it can be shown that an isotropic medium can give rise to negative refraction even if the permittivity and the permeability are both positive.<sup>21</sup> Such right-handed media are particularly promising for the observation of negative refraction and associated phenomena at optical frequencies, and we discuss their remarkable properties elsewhere.<sup>21</sup>

#### 5. LEFT-HANDED MATERIALS WITH $n > 0$

In the following we discuss the optical properties of propagating waves in type II and type III LHMs, i.e., LHMs that give rise to positive refraction. We do not consider evanescent waves; and to simplify the discussion, we neglect the imaginary parts of the permittivity and the permeability. Since the refractive indices for type II and type III LHMs are positive, it follows that a light beam that traverses an interface formed by a type II or type III LHM and an ordinary material will experience ordinary (positive) refraction. In Fig. 3 we show the boundary conditions for such an interface.

The two possible wave vectors of the refracted ray,  $\mathbf{k}_a$  and  $\mathbf{k}_b$ , both satisfy the boundary conditions. The correct solution can be identified only with recourse to Einstein causality (and not the direction of the Poynting vector). Since the LHM is assumed to be of type II or type III, its refractive index is positive and given by

$$n_2 = + \sqrt{\frac{\epsilon_2 \mu_2}{\epsilon_0 \mu_0}} > 0, \quad (11)$$

such that  $\mathbf{k}_a$  is unphysical and must be rejected.

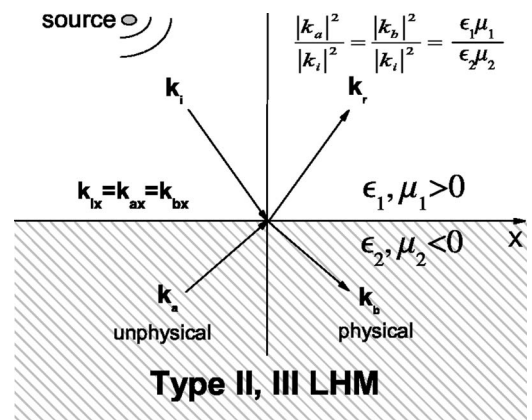


Fig. 3. Boundary conditions for an interface between an ordinary material ( $\epsilon_1, \mu_1 > 0$ ) and a type II or type III LHM ( $\epsilon_2, \mu_2 < 0$ ). Two conditions have to be met: (i) the transverse components of the wave vectors must be the same across the boundary ( $\mathbf{k}_{ix} = \mathbf{k}_{ax} = \mathbf{k}_{bx}$ ), and (ii) the ratio of the magnitudes of the transmitted wave vector to that of the incidence wave vector must satisfy  $(|\mathbf{k}_a|/|\mathbf{k}_i|)^2 = (|\mathbf{k}_b|/|\mathbf{k}_i|)^2 = (\epsilon_1 \mu_1 / \epsilon_2 \mu_2)$ .

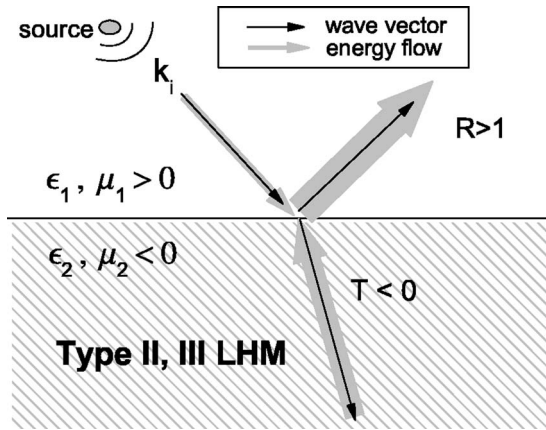


Fig. 4. Schematic of the energy flux across an interface between an ordinary material and a (type II or type III) LHM with a positive refractive index. Energy is conserved and the refraction is positive. The net energy derives from the inverted LHM.

It is interesting to consider reflection and transmission at the boundary depicted in Fig. 3. The Fresnel equations are readily derived. For an incident beam with  $\mathbf{E}$  perpendicular to the plane of incidence, we obtain

$$r_{\perp} \equiv \left( \frac{E_r}{E_i} \right)_{\perp} = \frac{\sin \theta_t \cos \theta_i + \frac{\mu_1}{|\mu_2|} \sin \theta_i \cos \theta_t}{\sin \theta_t \cos \theta_i - \frac{\mu_1}{|\mu_2|} \sin \theta_i \cos \theta_t},$$

$$t_{\perp} \equiv \left( \frac{E_t}{E_i} \right)_{\perp} = \frac{2 \sin \theta_t \cos \theta_i}{\sin \theta_t \cos \theta_i - \frac{\mu_1}{|\mu_2|} \sin \theta_i \cos \theta_t}. \quad (12)$$

Similarly, for the case of  $\mathbf{E}$  parallel to the plane of incidence, we have

$$r_{\parallel} \equiv \left( \frac{E_r}{E_i} \right)_{\parallel} = \frac{-\frac{\mu_1}{|\mu_2|} \sin \theta_i \cos \theta_t - \sin \theta_t \cos \theta_i}{\sin \theta_t \cos \theta_i - \frac{\mu_1}{|\mu_2|} \sin \theta_i \cos \theta_t}$$

$$t_{\parallel} \equiv \left( \frac{E_t}{E_i} \right)_{\parallel} = \frac{2 \sin \theta_t \cos \theta_i}{\sin \theta_t \cos \theta_i - \frac{\mu_1}{|\mu_2|} \sin \theta_i \cos \theta_t}, \quad (13)$$

where  $\theta_i$  is the angle of incidence and  $\theta_t$  is the angle of refraction. The reflectance  $R$  and transmittance  $T$  are correspondingly given by

$$R_{\perp} = r_{\perp}^2$$

$$T_{\perp} = - \left( \frac{\mu_1}{|\mu_2|} \right) \left( \frac{n_2}{n_1} \right) \left( \frac{\cos \theta_t}{\cos \theta_i} \right) t_{\perp}^2,$$

$$R_{\parallel} = r_{\parallel}^2,$$

$$T_{\parallel} = - \left( \frac{\mu_1}{|\mu_2|} \right) \left( \frac{n_2}{n_1} \right) \left( \frac{\cos \theta_t}{\cos \theta_i} \right) t_{\parallel}^2, \quad (14)$$

where the refractive index of the ordinary material  $n_1$  is

$$n_1 = + \sqrt{\frac{\epsilon_1 \mu_1}{\epsilon_0 \mu_0}} > 0. \quad (15)$$

Energy conservation across the boundary holds as Eqs. (14) satisfy

$$R_{\perp} + T_{\perp} = R_{\parallel} + T_{\parallel} = 1. \quad (16)$$

This is to be expected, as any solution (including both the retarded and the advanced solution) that satisfies Maxwell's equations will necessarily also satisfy Poynting's equation:

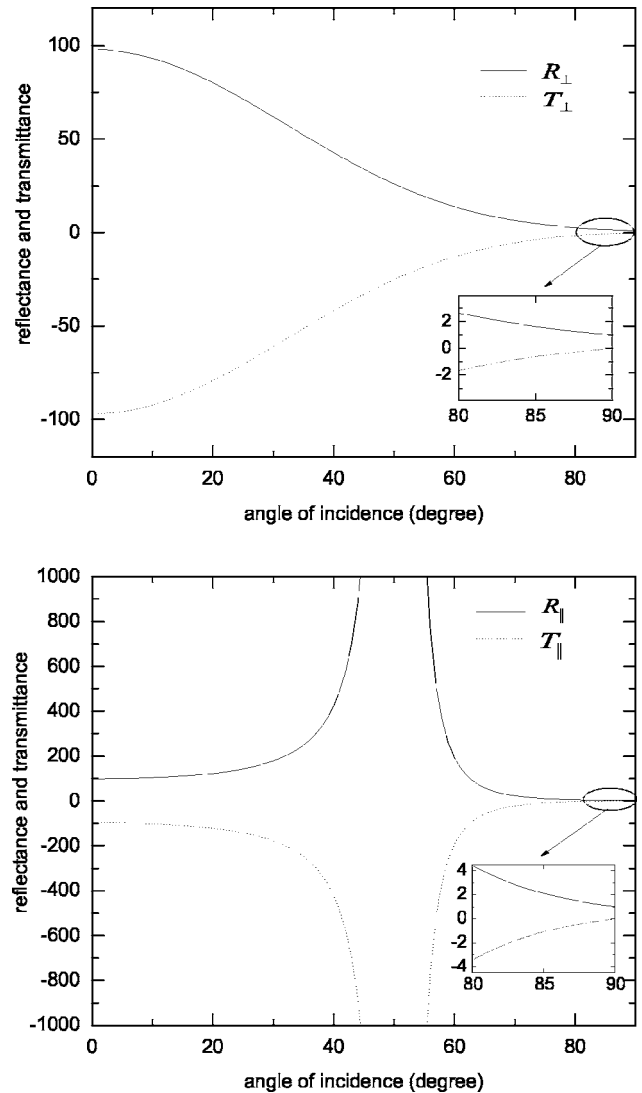


Fig. 5. Reflectance and transmission curves for electric field polarization perpendicular to the plane of incidence (top panel) and electric field polarization parallel to the plane of incidence (bottom panel).

$$\frac{\partial u}{\partial t} + \nabla \cdot \mathbf{S} = 0, \quad (17)$$

where  $\mathbf{S}$  is the Poynting vector (the energy flux) and  $u$  is the energy density.

However, it is surprising that  $T$  is negative at such a boundary [see Eqs. (14)]. It follows that energy is transported across the boundary out of the LHM. The energy flux thus points in the direction opposite to the wave vector of the transmitted wave (as is the case for all types of LHM where  $\mathbf{S}$  is opposite to  $\mathbf{k}$ , regardless of the sign of their refractive indices). This is schematically shown in Fig. 4.

Furthermore, since energy is conserved ( $R+T=1$ ), it follows that the reflectance  $R$  must be greater than unity. The energy needed to amplify the reflected beam is derived from the LHM, which is assumed to have gain and therefore must have been pumped. Enhanced reflection from an amplifying medium is not restricted to a LHM. Cyburski and Silverman have observed reflectances of 2 to 3 from an optically pumped gain medium.<sup>22,23</sup>

Figure 5 shows reflectance and transmittance curves for the boundary of a left-handed gain medium with an ordinary material. We take  $\mu_2 = -\mu_1$  and  $\epsilon_2 = -1.5\epsilon_1$ . Anomalous high  $R_{\parallel}$  and  $T_{\parallel}$  are predicted near an angle of incidence  $\theta_i \sim 50^\circ$ . From Eqs. (12) one can see that the angle for which  $R_{\parallel}$  and  $T_{\parallel}$  become infinite would correspond to Brewster's angle for an interface where the second material's permittivity and permeability have the same magnitude but are of the opposite (positive) sign. The singularity is clearly unphysical and is a consequence of unrealistic assumptions, such as a constant gain of the inverted medium<sup>22</sup> and neglect of imaginary parts of permeability and permittivity. Inclusion of the imaginary parts will remove the infinity of  $r_{\parallel}$  and  $t_{\parallel}$  in Eqs. (13) that leads to the singularity in Fig. 5.<sup>24</sup> Nevertheless, it is clear that enhanced reflection can be expected at the boundary of a LHM with gain.

## 6. CONCLUSIONS

We discuss how the sign of the refractive index in any medium may be rigorously derived using a theory of causal wave propagation.<sup>7,8,11,15</sup> In particular, we consider the connection between Einstein causality and the real part of the refractive index for media that have negative permittivities and permeabilities (LHM). We find that isotropic LHMs that are in their ground state can exhibit negative refraction, whereas LHMs that have gain, i.e., are (partially) inverted, do not necessarily have a negative index of refraction. We introduce four different types of LHM and predict that two of them will give rise to positive refraction. We also discuss reflection at the boundary of a left-handed gain medium.

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24. When the denominator in Eqs. (13)  $\rightarrow 0$ , then  $r_{\parallel}$  and  $t_{\parallel} \rightarrow \infty$ . To determine the angle of incidence  $\theta_i$  at which this occurs, note that the denominator can be expressed as  $[(n_1 \sin \theta_i)/n_2][1 - (n_1^2/n_2^2)\sin^2 \theta_i]^{1/2} - (\mu_1/|\mu_2|)\sin \theta_i \cos \theta_i$ . For the example considered in the text where  $\mu_2 = -\mu_1$  and  $\mu_1 > 0$ , the denominator vanishes when  $(n_1/n_2) \times [1 - (n_1^2/n_2^2)\sin^2 \theta_i]^{1/2} = \cos \theta_i$ . This yields  $[(n_1^2 - n_2^2)/n_2^2] \sin^2 \theta_i = [(n_1^2 - n_2^2)/n_1^2] \cos^2 \theta_i$ , and the singularity occurs when  $\tan \theta_i = (n_2/n_1)$ . It then follows that the inclusion of the imaginary part of  $\epsilon_2$  (which yields a complex  $n_2$ ) removes the singularity, since this relation can no longer be satisfied for a real angle of incidence  $\theta_i$ .